Starter Question

Rewrite this expression in terms of $\log x$, $\log y$ and $\log z$:

$$\log \frac{x^2\sqrt{y}}{z^4}$$

$$¿\,2\log x + \frac{1}{2}\log y - 4\log z$$

Solve $1 + 2\log_3 x = \log_3 (5x + 2)$.

Hint:

$$x = 2\,,\, as\, x \neq -\frac{1}{3}$$

Q Kinematics

Q1

Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.

Assessed at AS and A-level

Teaching guidance

Students should be able to:

- understand positions described relative to a given origin
- understand and describe the position of a particle through a combination of its initial position and a displacement
- demonstrate an understanding of the relationship between the vector quantities displacement and velocity and their associated scalar quantities distance and speed
- understand average speed and average velocity.

Q2	Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph.

Assessed at AS and A-level

Teaching guidance

Students should be able to:

- use the gradient of a displacement-time graph to find the velocity (or speed)
- use the gradient of a velocity-time graph to find the acceleration and interpret positive and negative gradients
- understand that graphs may include negative velocities
- use the area under a velocity-time graph to find displacement
- sketch either a displacement-time or velocity-time graph for a given scenario.

7.2 Motion in a straight line

<u>Definitions</u>

Position □ *a vector*: the distance and direction from the origin O.

Displacement □ *a vector*: the change of position

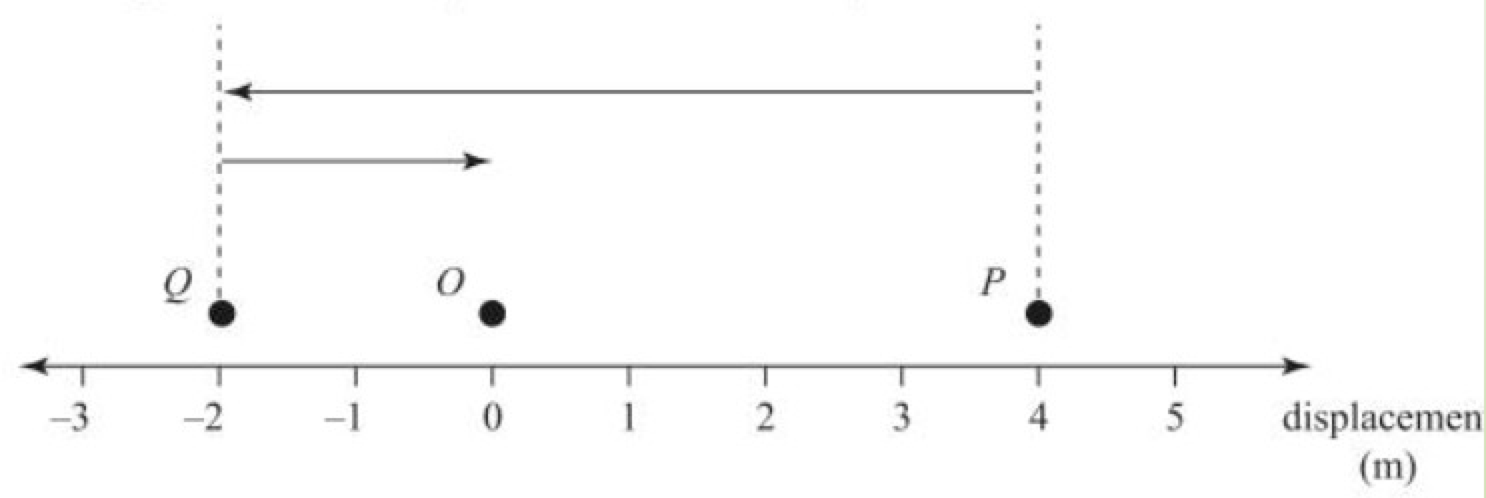
Distance □ *a scalar*: the magnitude of the displacement (provided there is no change of direction when moving position)

Velocity □ *a vector*: the rate of change of displacement

Speed □ *a scalar*: the magnitude of velocity

7.2 Motion in a straight line

It is important to distinguish between displacement and distance. Consider the follow



The diagram shows displacement = -6 from position 4 to position -2, then displacement = 2 from position -2 to position 0 (the origin). The resultant displacement is

7.2 Motion in a straight line

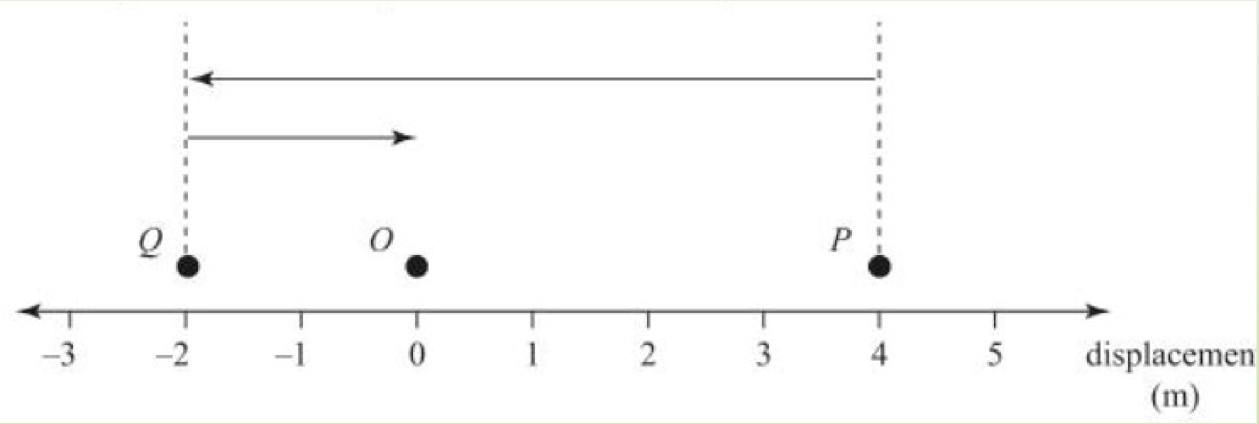
Similarly, there is a difference between average velocity and average speed. Average speed does not take into account the direction of motion. Hence:

$$\boxed{Average\ Velocity = \frac{Resultant\ Displacement}{Total\ Time}}$$

$$\boxed{Average\ Speed = \frac{Total\ Distance}{Total\ Time}}$$

7.2 Motion in a straight line

So, using the previous scenario of a particle travelling from P Q O in a total time of 10 seconds.

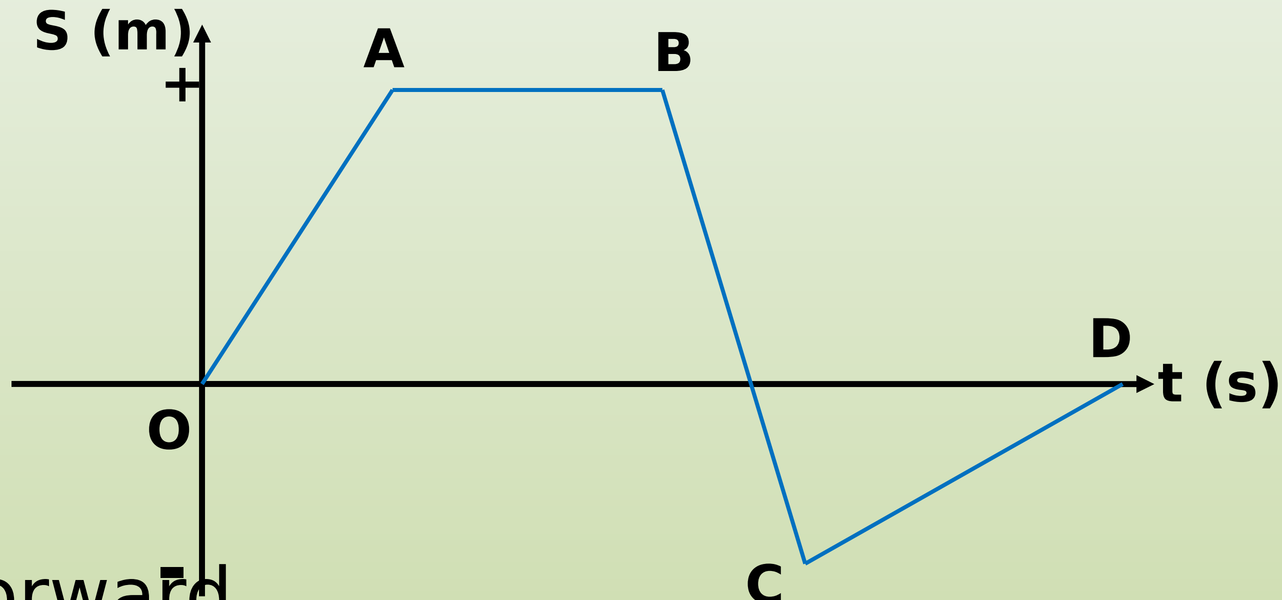


Resultant displacement = -4m

Total distance = 8m

7.2 Motion in a straight line

Displacement – Time (s-t) Graphs



OA → moving forward

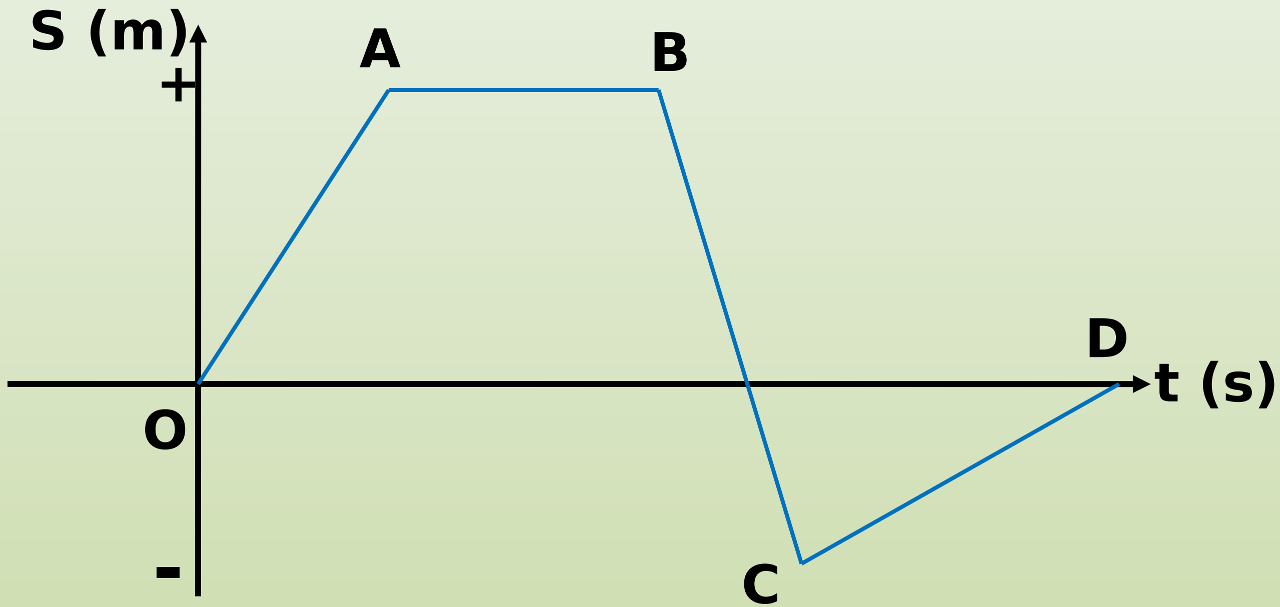
AB → stationary

BC → moving backwards (faster than forward)

CD → moving forward (slower than OA)

7.2 Motion in a straight line

Displacement – Time (s-t) Graphs

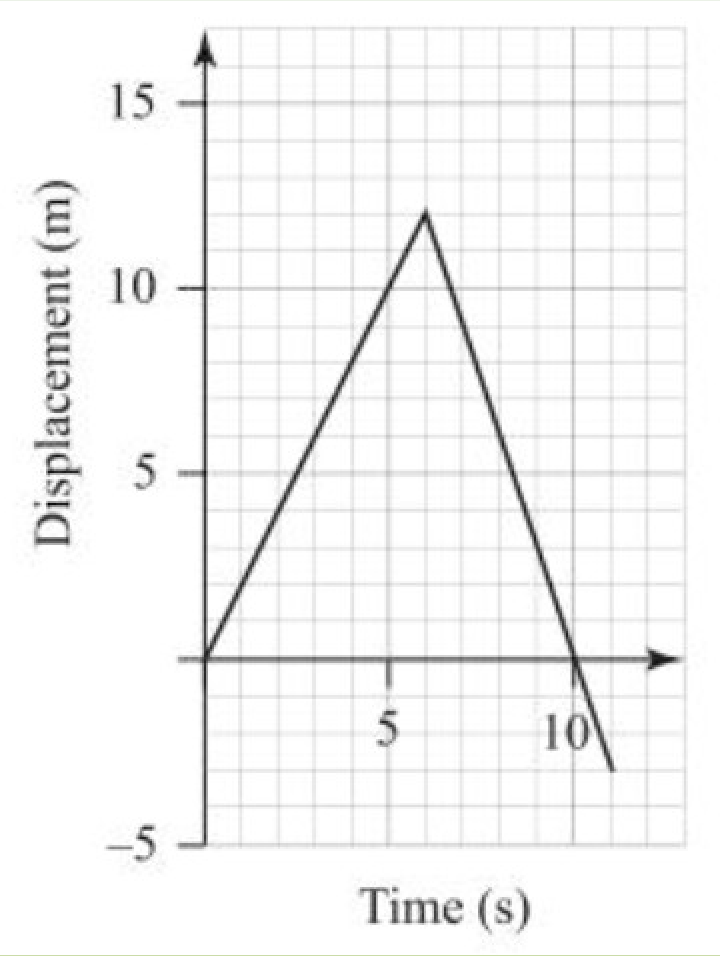


7.2 Motion in a straight line

Example 1

The graph shows the motion of a particle along a straight line between 0 and 11 seconds. Find:

a) the displacement and velocity for the first 6 seconds and the final 5 seconds



First 6s: displacement

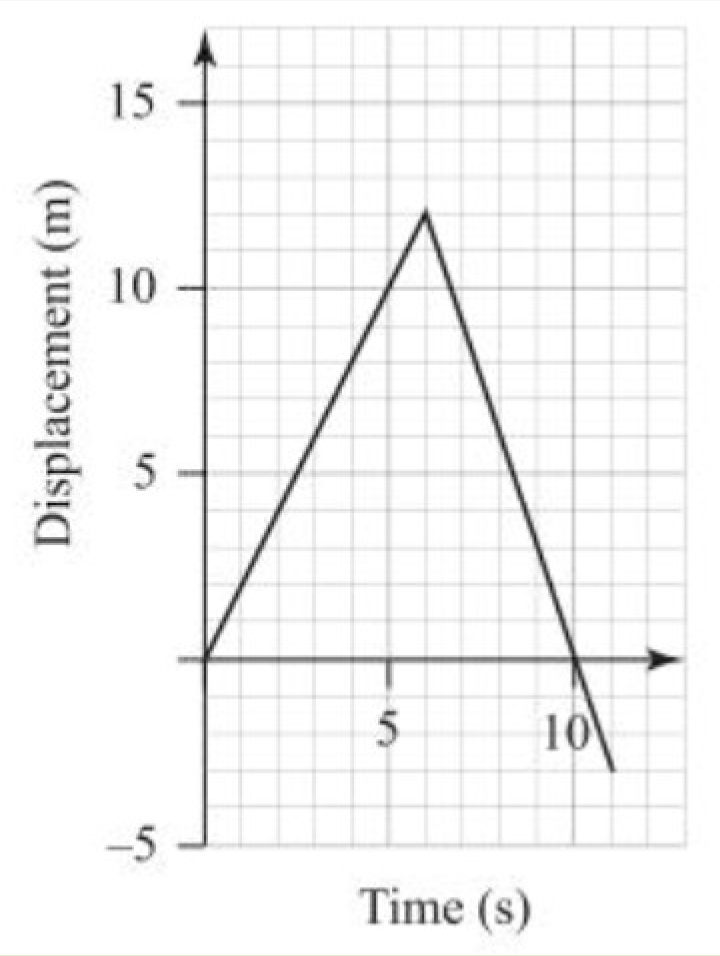
velocity

7.2 Motion in a straight line

Example 1

The graph shows the motion of a particle along a straight line between 0 and 11 seconds. Find:

a) the displacement and velocity for the first 6 seconds and the final 5 seconds



Final 5s: displacement

velocity

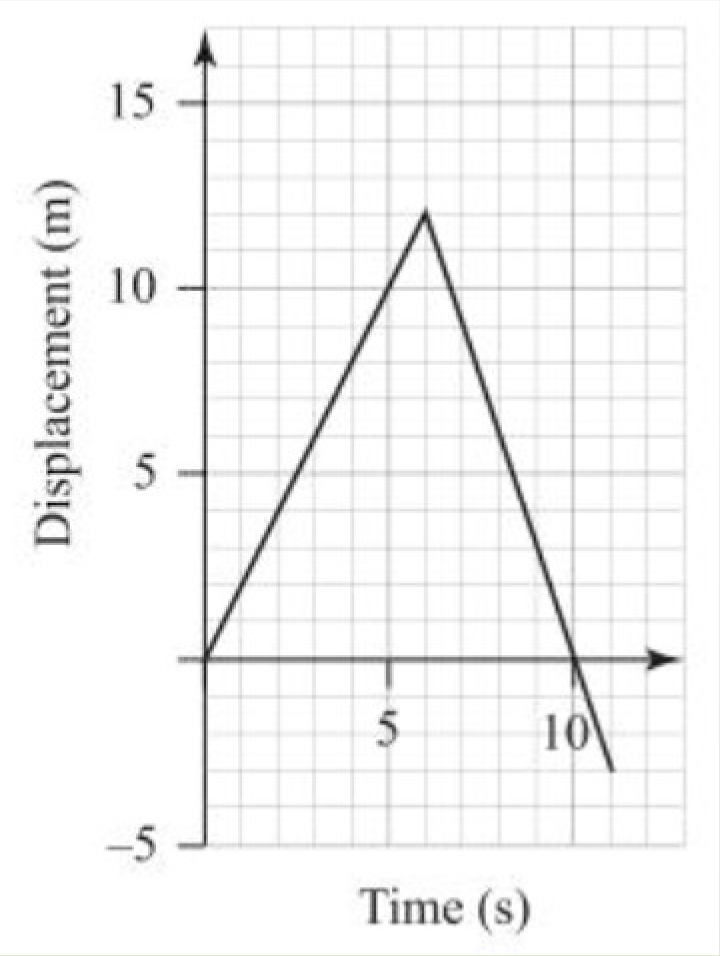
7.2 Motion in a straight line

Example 1

The graph shows the motion of a particle along a straight line between 0 and 11 seconds. Find:

b) (i) the resultant displacement

resultant displacement



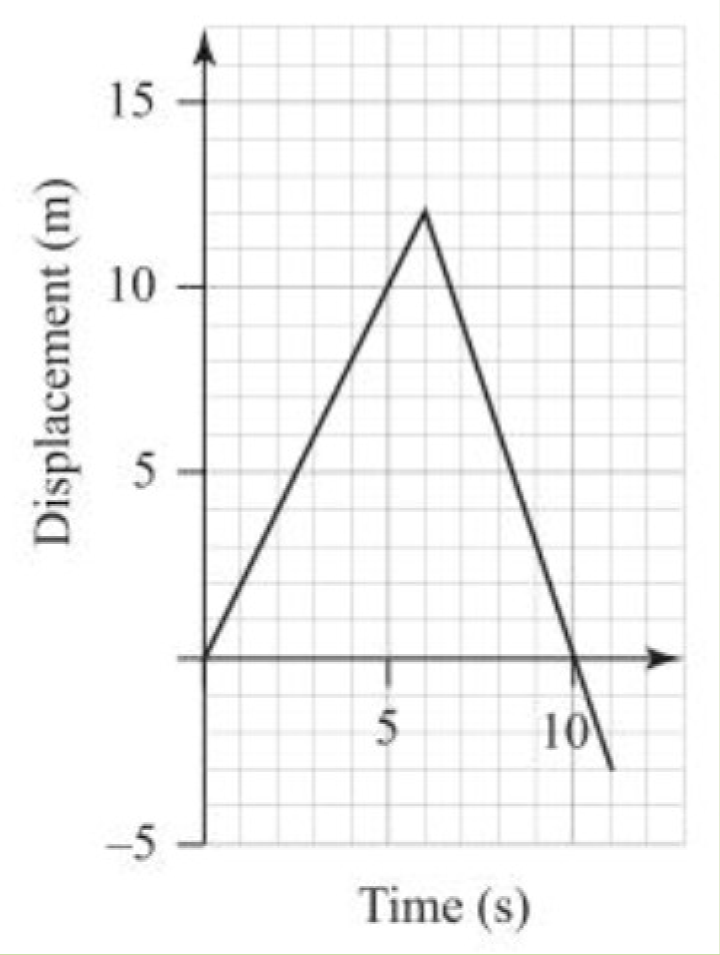
7.2 Motion in a straight line

Example 1

The graph shows the motion of a particle along a straight line between 0 and 11 seconds. Find:

b) (ii) the average velocity

average velocity



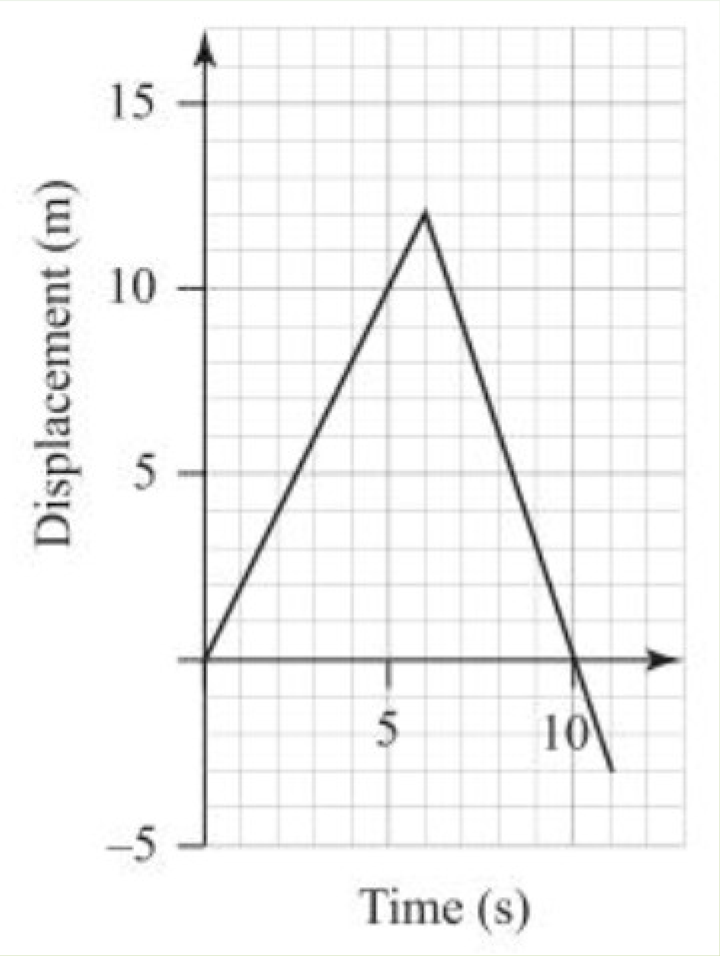
7.2 Motion in a straight line

Example 1

The graph shows the motion of a particle along a straight line between 0 and 11 seconds. Find:

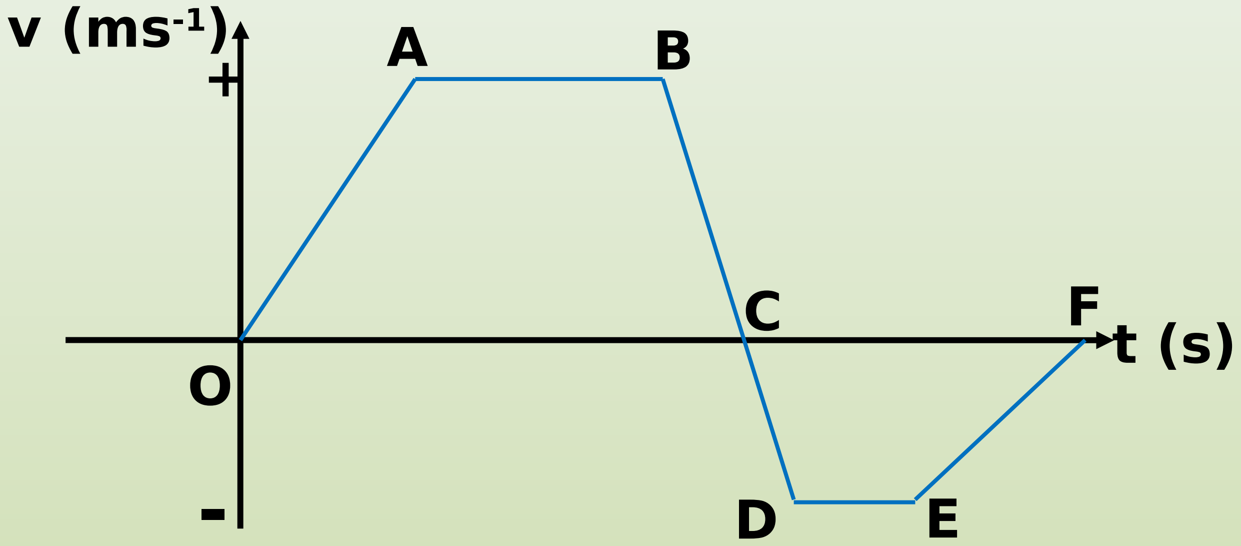
b) (iii) the average speed

average speed



7.2 Motion in a straight line

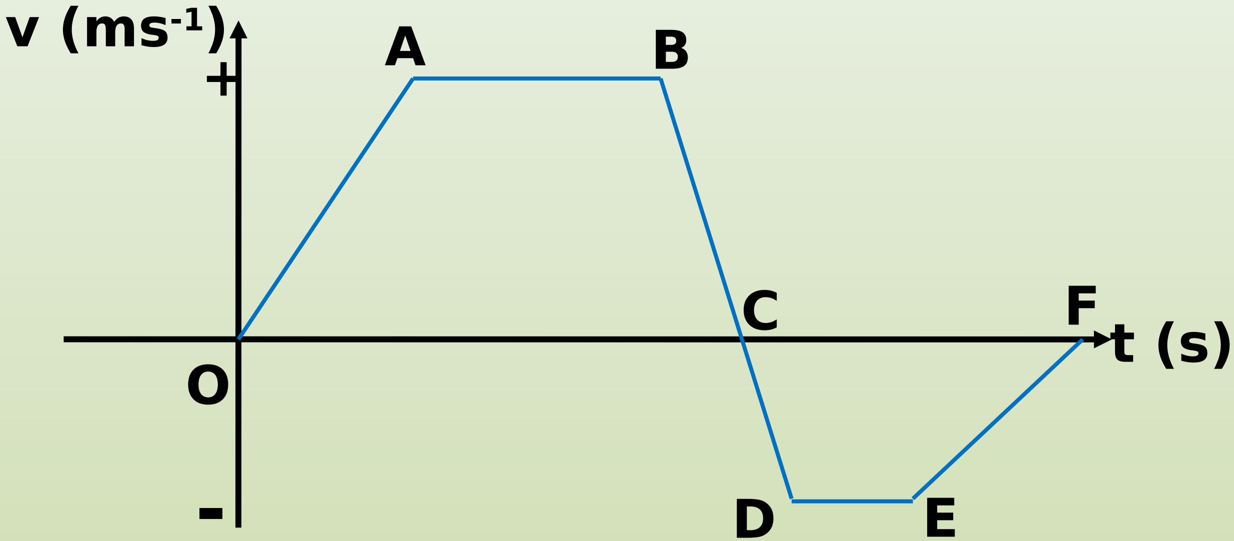
Velocity – Time (v-t) Graphs



OA increasing velocity, forwards
AB constant velocity, forwards
BC decreasing velocity, forwards
CD increasing velocity, backwards
DE constant velocity, backwards
EF decreasing velocity, backwards

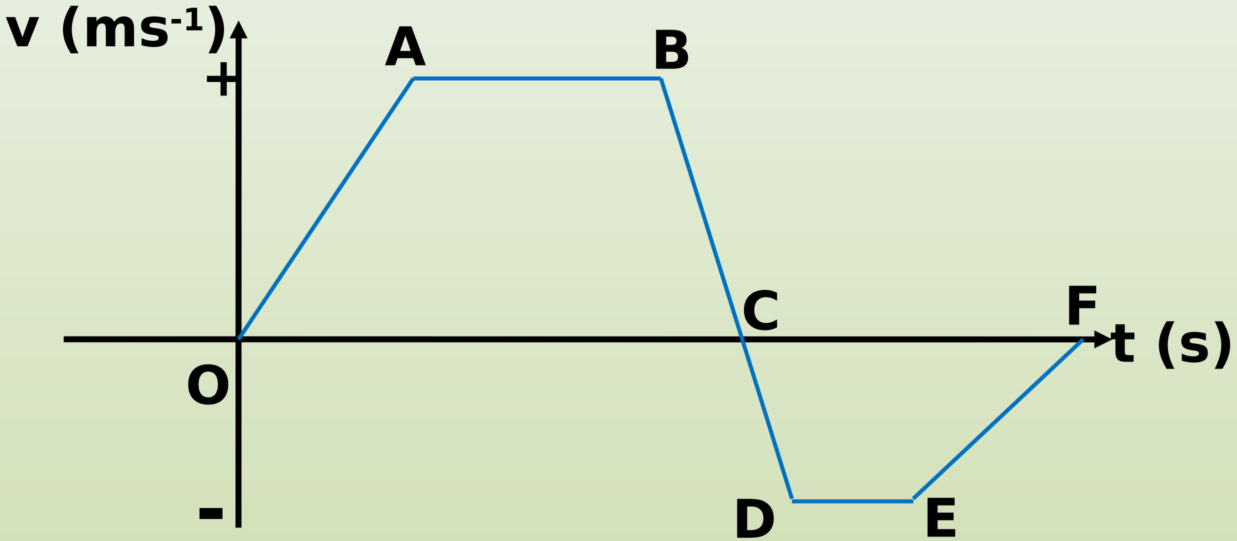
7.2 Motion in a straight line

Velocity – Time (v-t) Graphs



7.2 Motion in a straight line

Velocity – Time (v-t) Graphs

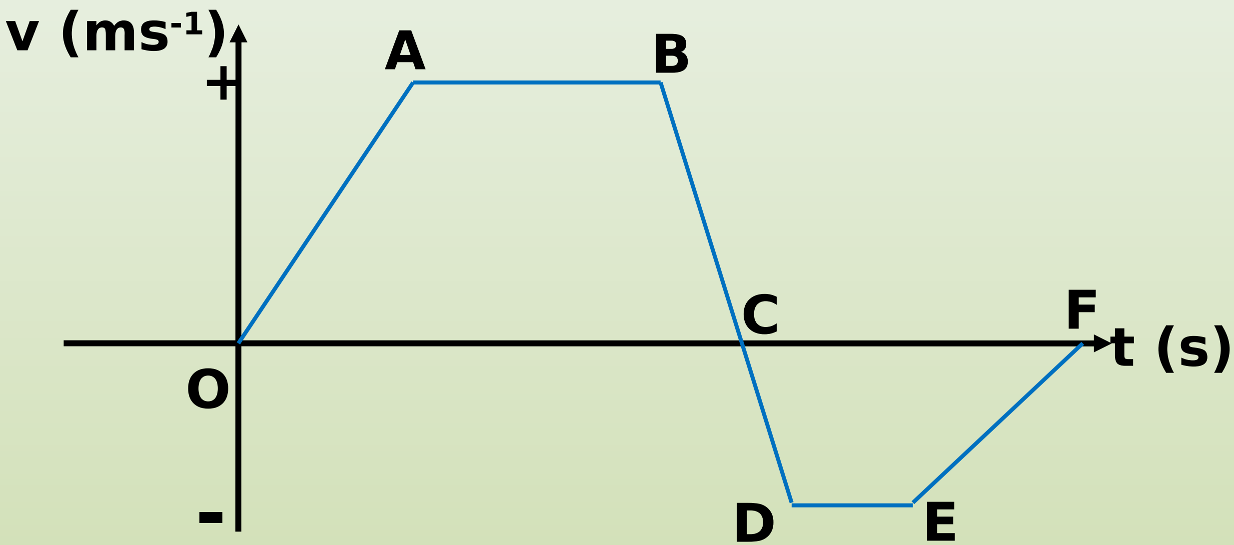


If velocity changes from **ms^{-1}** to **ms^{-1}** in **t seconds**, then:

acceleration rate of change of velocity

7.2 Motion in a straight line

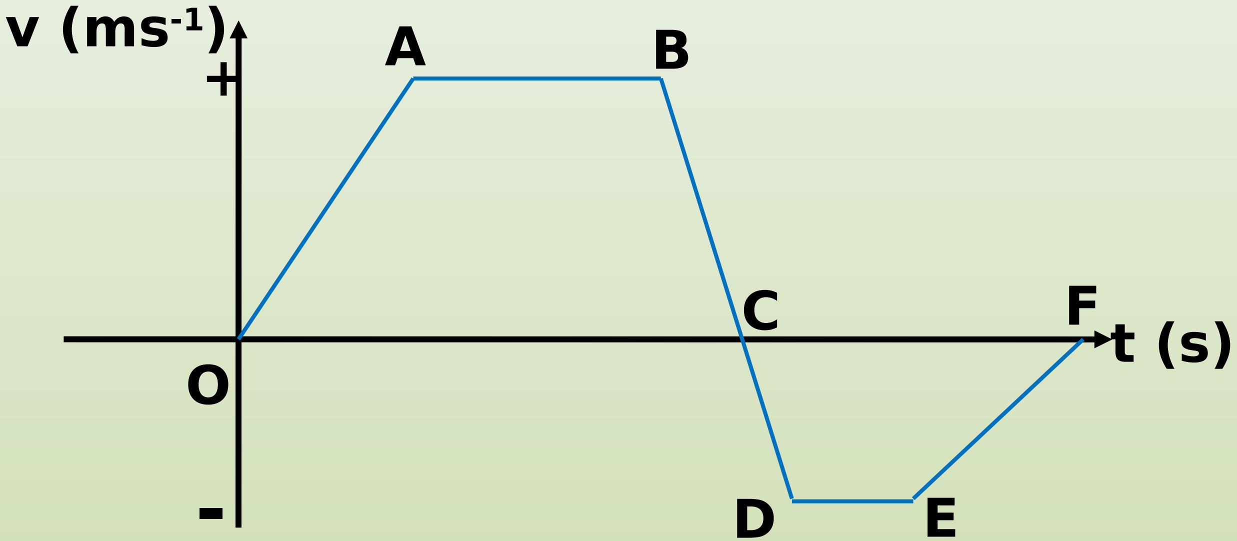
Velocity – Time (v-t) Graphs



A negative acceleration is often referred to as a deceleration. In the graph above, OA is accelerating and BC is decelerating.

7.2 Motion in a straight line

Velocity – Time (v-t) Graphs



The area between the - graph and the -axis is the

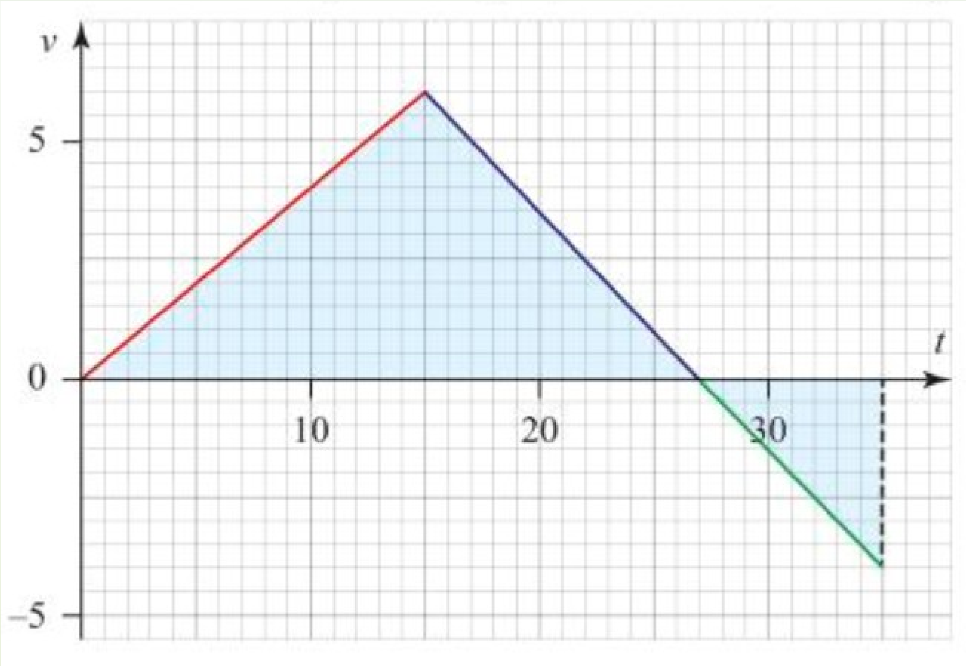
displacement

7.2 Motion in a straight line

Example 2

a) Calculate the acceleration during:

i. the first 15 seconds



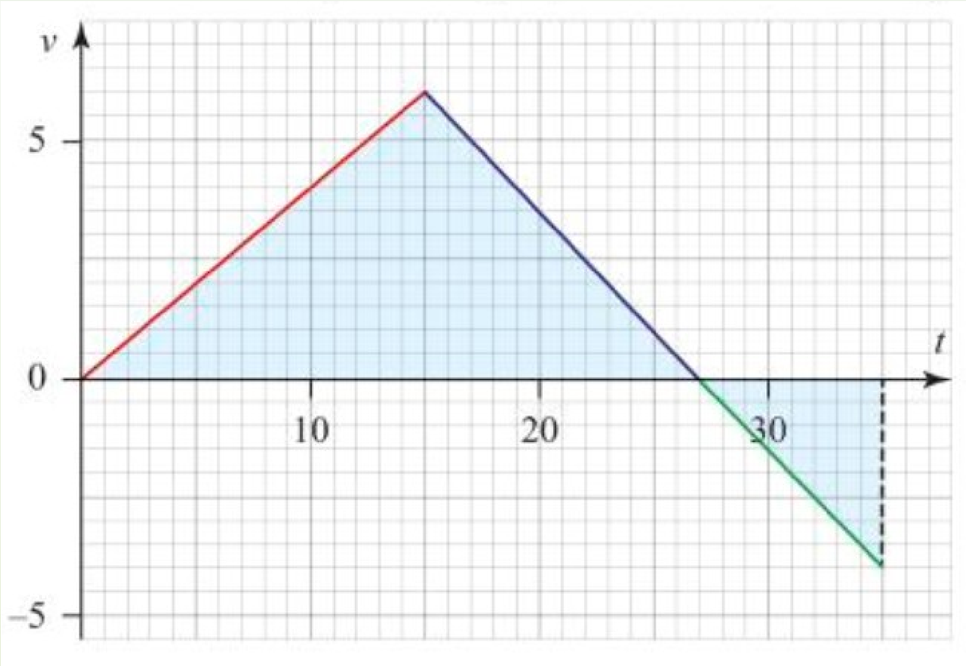
Acceleration = $\frac{\text{change in velocity}}{\text{change in time}}$

7.2 Motion in a straight line

Example 2

a) Calculate the acceleration during:

ii. the remaining 20s



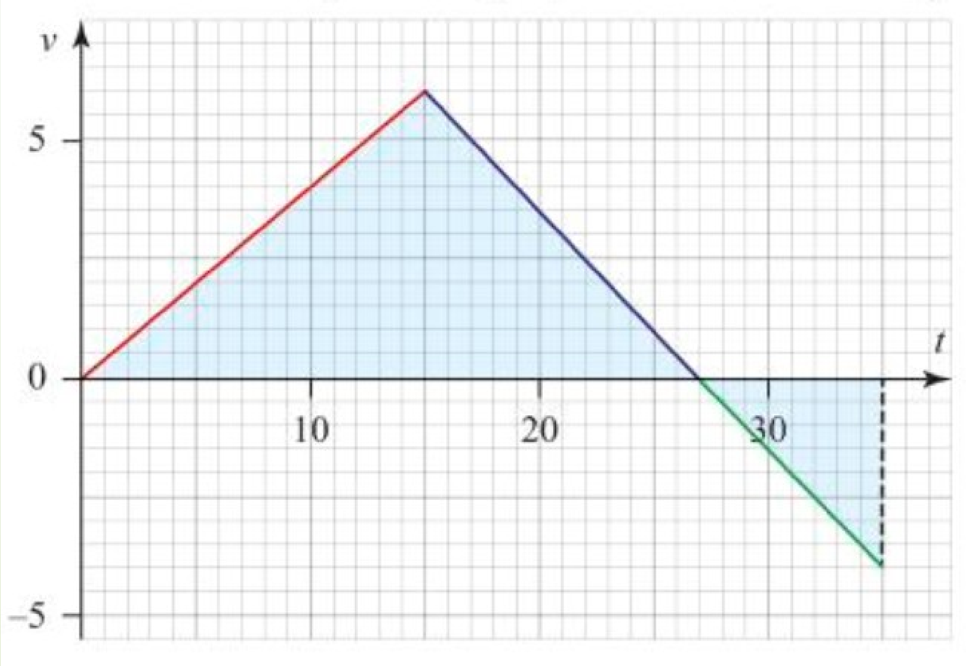
$$a = \frac{-4-6}{35-15} = -\frac{10}{20} = -0.5\,ms^{-2}$$

7.2 Motion in a straight line

Example 2

b) Calculate:

i. the resultant displacement



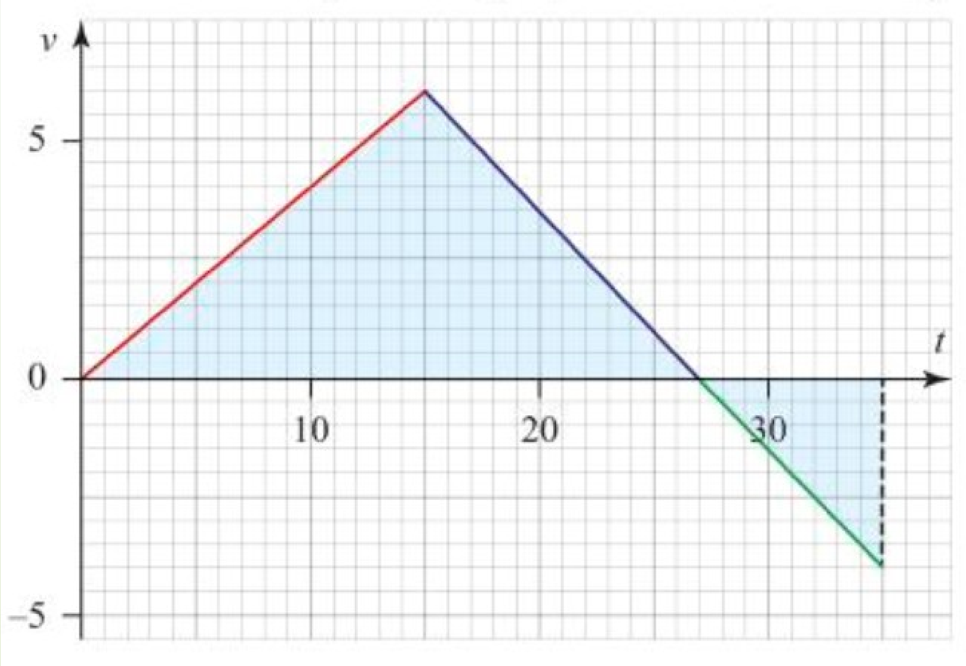
In the first 27s, the object is moving forward.
Displacement = area under the graph
= ½ base x height

7.2 Motion in a straight line

Example 2

b) Calculate:

i. the resultant displacement



In the last 8s, the object moves backwards:
Displacement = ½ base x height

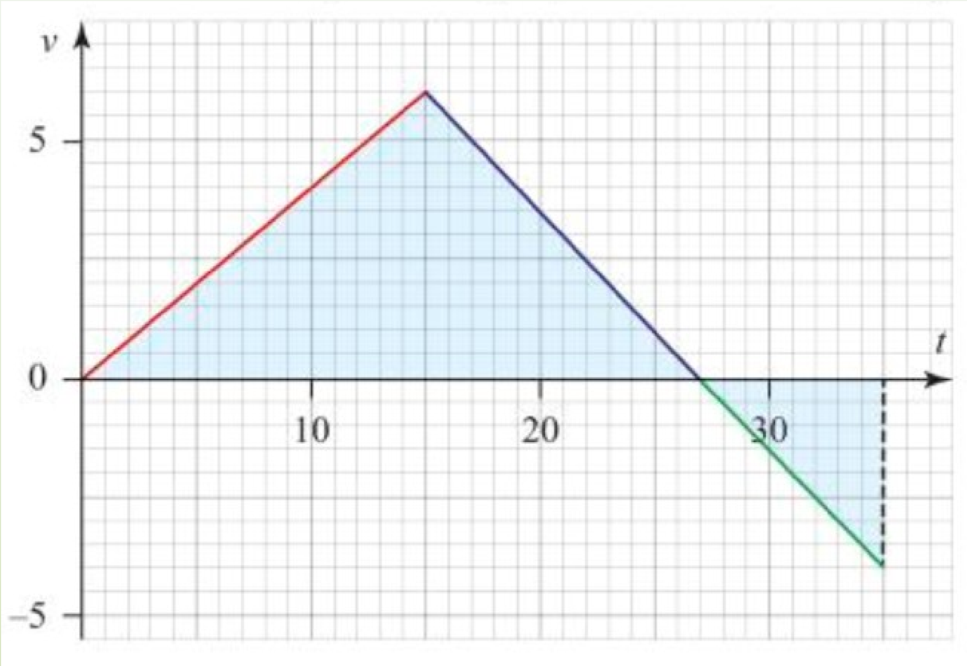
resultant displacement

7.2 Motion in a straight line

Example 2

b) Calculate:

ii. the total distance travelled



From part b) i)
Distance forward = 81m
Distance backward = 16m
total distance travelled = 81 + 16 = 97m

Example 3

A van moves from rest on a straight horizontal road.

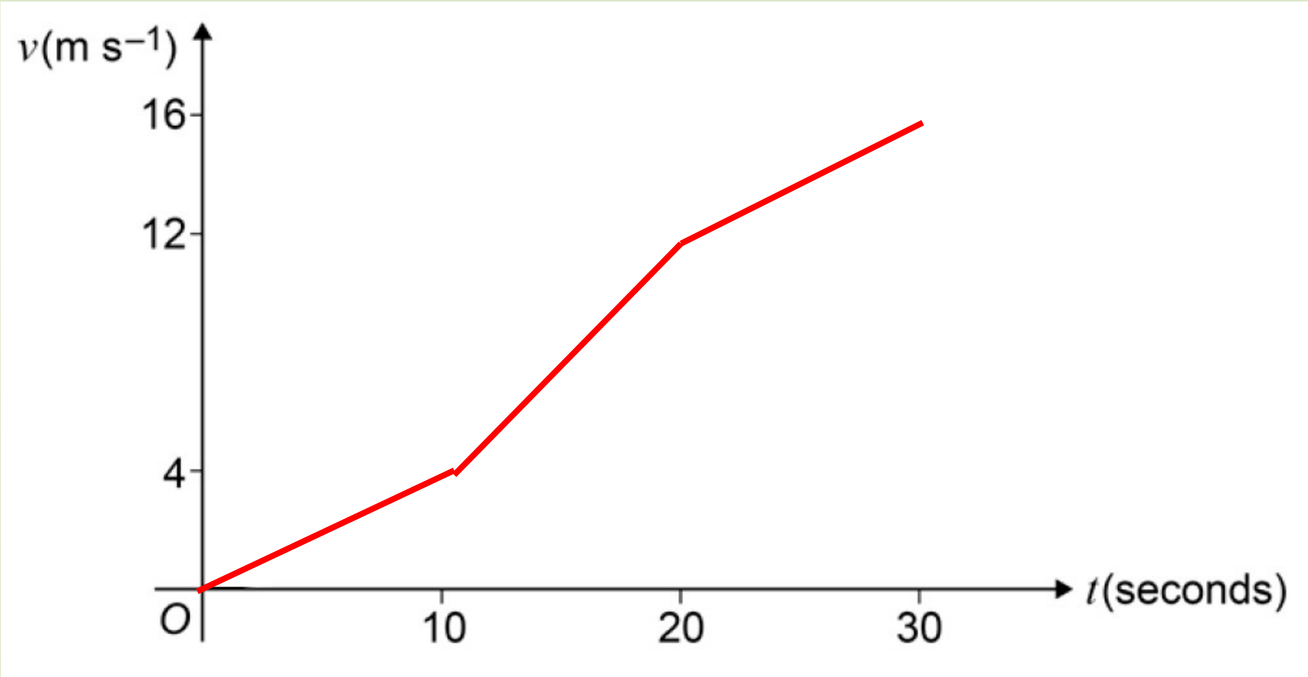
(a) In a simple model, the first 30 seconds of the motion are represented by three separate stages, each lasting 10 seconds and each with a constant acceleration.

During the first stage, the van accelerates from rest to a velocity of 4 m s^{-1}

During the second stage, the van accelerates from 4 m s^{-1} to 12 m s^{-1}

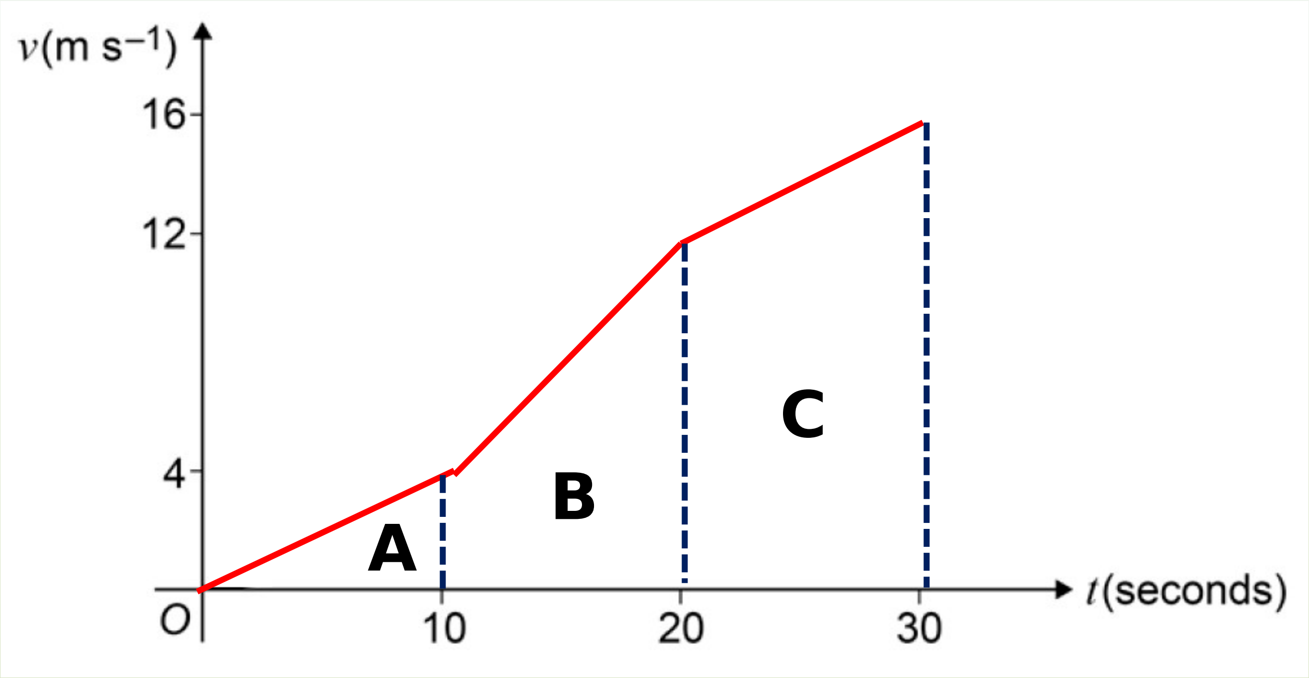
During the third stage, the van accelerates from 12 m s^{-1} to 16 m s^{-1}

(i) Sketch a velocity-time graph to represent the motion of the van during the first 30 seconds of its motion.



Example 3

(ii) Find the total distance that the van travels during the 30 seconds.



Area A =

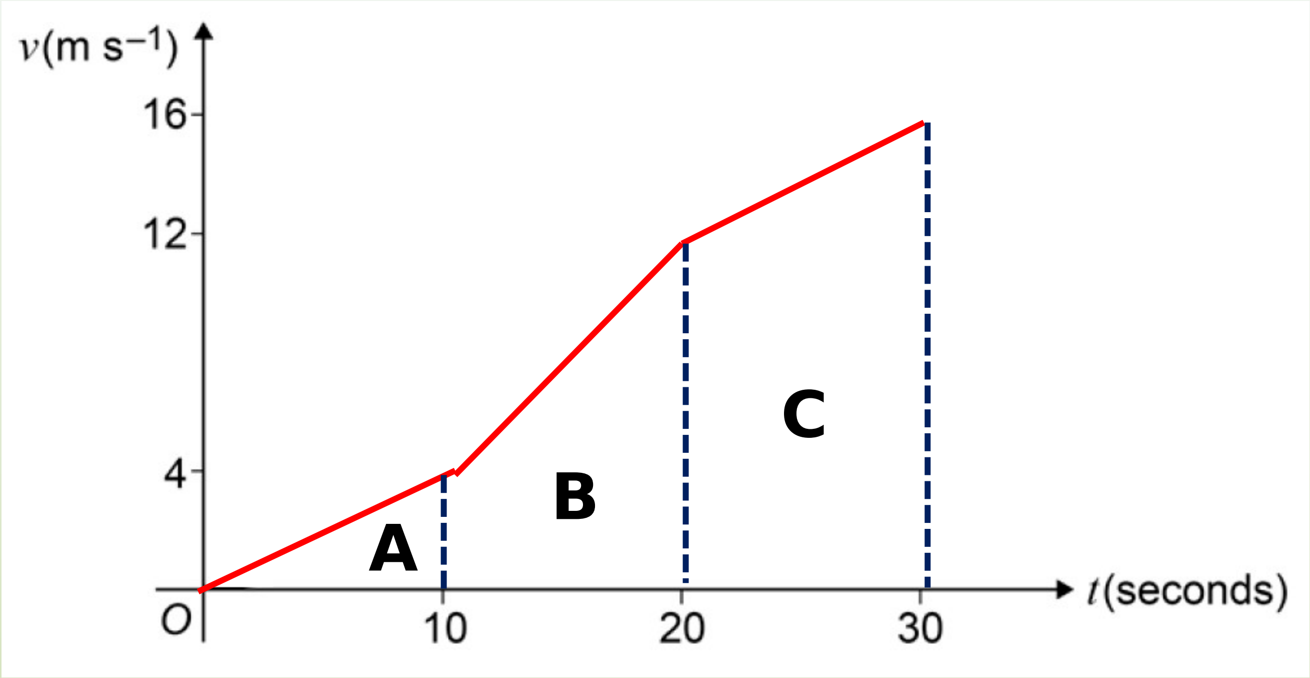
Area B =

Area B =

total distance travelled

Example 3

(iii) Find the greatest acceleration of the van during the 30 seconds.



eatest acceleration is the greatest gradient.
s occurs between 10 and 20 seconds.

7.2 Motion in a straight line

Example 4a

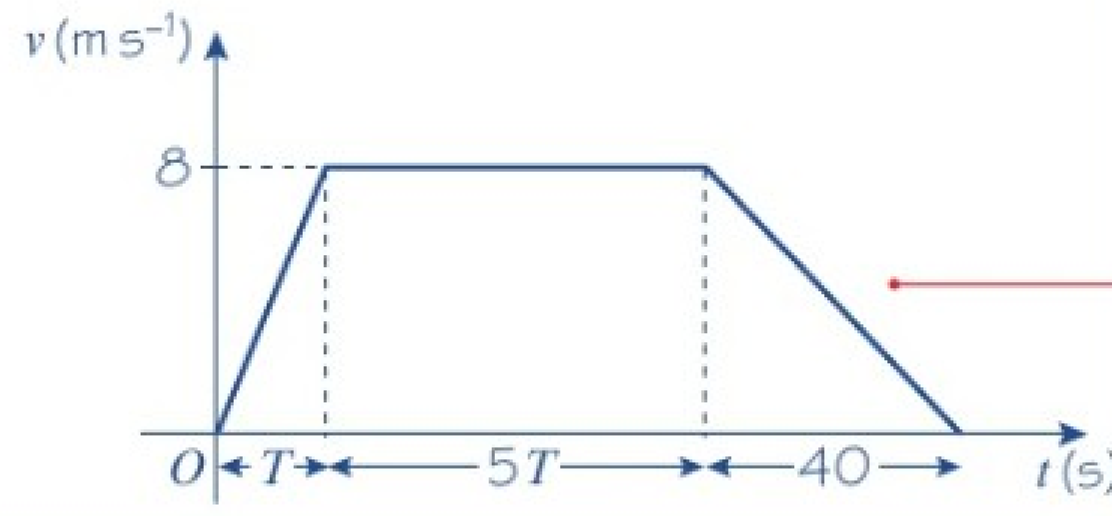
A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 8 m s^{-1} in T seconds. The particle then travels at a constant velocity of 8 m s^{-1} for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s.

a Sketch a velocity–time graph to illustrate the motion of the particle.

Given that the total displacement of the particle is 600 m:

b find the value of T.

a



7.2 Motion in a straight line

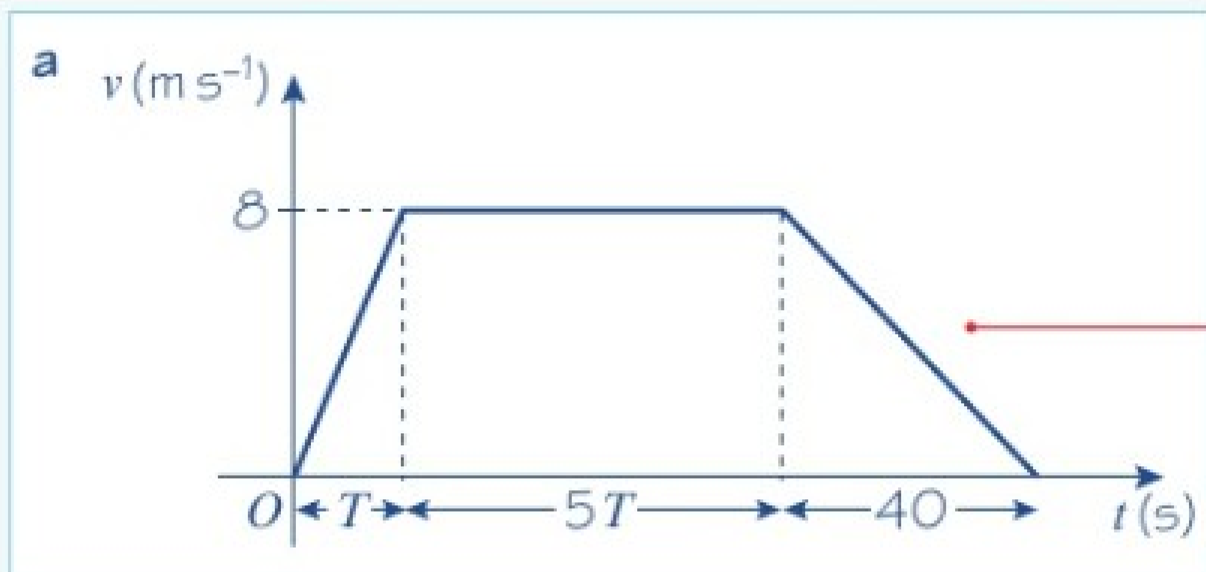
Example 4b

A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 8 m s^{-1} in T seconds. The particle then travels at a constant velocity of 8 m s^{-1} for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s.

a Sketch a velocity–time graph to illustrate the motion of the particle.

Given that the total displacement of the particle is 600 m:

b find the value of T.



7.2 Motion in a straight line

Example 4b

A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of $8\ \mathrm{m\,s^{-1}}$ in T seconds. The particle then travels at a constant velocity of $8\ \mathrm{m\,s^{-1}}$ for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s.

a Sketch a velocity–time graph to illustrate the motion of the particle.

Given that the total displacement of the particle is 600 m:

b find the value of T.

7.2 Motion in a straight line

Easier:
- June 2008
- Jan 2011
- June 2013

Medium:
- Jan 2006
- Jan 2007
- Jan 2009
- June 2010
- Jan

Most challenging:
- Integral Qs
- Challenge Q

7.2 Motion in a straight line

hallenge Question

5 A car is initially at rest. On a short journey the car

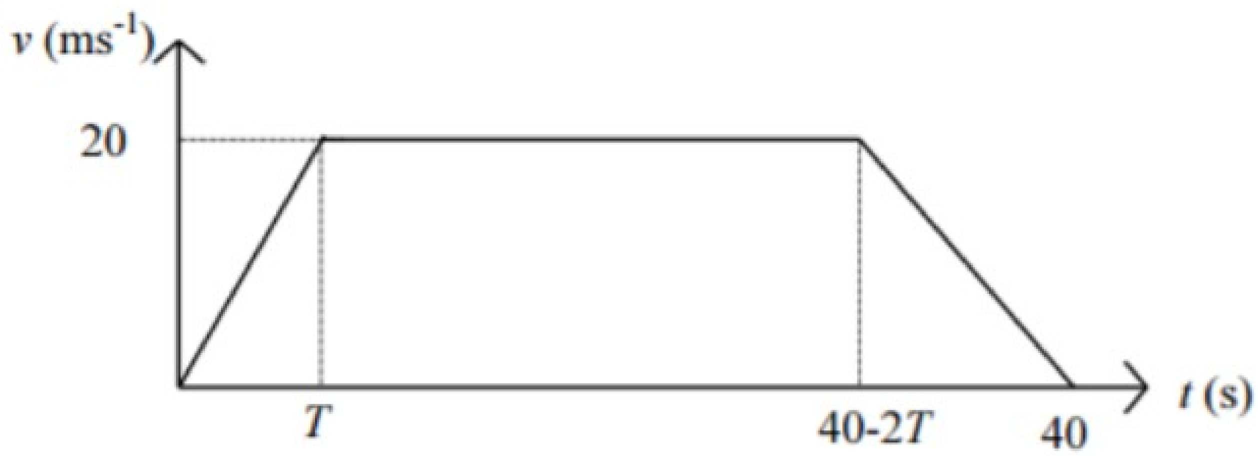
I. accelerates uniformly for T seconds to a speed of 20 ms^{-1},
II. then travels at this speed for a period of time,
III. then decelerates uniformly for $2T$ seconds before coming to rest.

(a) In one journey the car moves for a total of 40 seconds and travels a total of 620 m. Using this information:

sketch a velocity-time graph and hence, or otherwise, find T; *(5 marks)*

(b) In the case when $T = 5$, find the time that it would take the car to complete a 1000 m journey. *(3 marks)*

7.2 Motion in a straight line

5 (a)(i)		B1 B1		
	$620 = \frac{1}{2}\times(40+(40-3T))\times 20$	M1A1		M1: use of area under graph to find T
	$620 = 800 - 30T$			
	$T = 6$ seconds	A1	(5)	

(b)	$1000 = \frac{1}{2}\times\left(t+(t-15)\right)\times 20$	M1 A1		M1: forming equation for t based on area under graph
	$100 = 2t - 15$	A1	(3)	
	$t = 57.5$ seconds			